Analysis of the Integrate and Fire Model of a Neuron: With a Fixed Input Stimulus
BIPASHA DAS¹, ADITYA GAURAV²

Abstract: The integrate and fire model is the simplest and most widely used single neuron model for modeling neurons at an abstract level. It is one of the best known formal spiking neuron models. Simulation of integrate and fire model is done either through external current or in terms of synaptic inputs obtained from the presynaptic neurons. The synaptic input is considered to be stochastic with Gaussian white noise distribution. The integrate and fire model is a single compartment model which is useful for understanding how an ensemble of neurons, neighboring neurons which group together and fire together, interact and sum large number of synaptic inputs. When a threshold value of the membrane potential of a neuron is reached, firing occurs. A degree of variability exists in the firing output of the integrate and fire neurons when they receive synaptic input[1]. This review gives a brief overview of the integrate and fire model of the neuron and the mathematical techniques, such as stochastic differential equations that are available for analyzing the spike generation in this model.

Keywords: Integrate And Fire Model, Neuron Model, Spike Generation, Synaptic Input.

I. INTRODUCTION
The Integrate and fire model was first modelled by Lapicque (1907) when nothing specific was known about the biophysics of neuronal action potential and excitable membrane. Lapicque put forward a model of the neuron in terms of an electric circuit consisting of a capacitor and a resistor in parallel. In this simple deterministic model, the membrane capacitor was charged to a certain threshold potential. This generated an action potential and the capacitor would discharge resetting the potential. Lapicque calculated the firing rate of a nerve fibre that was resistively coupled to a fixed voltage stimulating electrode. At present, the integrate and fire model is not restricted to the simple capacitor-resistor circuit but also allows us to effectively model synaptic and sub-threshold conductance’s. This model has also been useful in the time scale separation between the stereotypical rapid voltage change during spike generation and relatively slow subthreshold integration. The integrate and fire model has been widely used in understanding the information processing capacities of a neuron. Studies of synaptic integration by single neurons or simulation of network of thousands of neurons are based on the integrate and fire model of the neuron [1,2].

The arrival time of synaptic inputs is apparently random. Gerstein and Mandelbrot(1964) formulated the earliest solution of the integrate and fire model by incorporating the stochastic activity modeled as a random walk for the incoming postsynaptic potentials[3]. Further development on this model are built upon this diffusion approach using stochastic differential equations and Ornstein-Uhlenback process (Uhlenback and Ornstein 1930). Stein included the decay of the membrane potential with stochastic input in the integrate and fire model. This paper deals with four sections. After a brief introduction in section 1, section 2 deals with the neuronal model in consequence of our study. Simulation of the integrate and fire model is shown in section 3. Section 4 discusses the results and conclusions of our study.

II. THE INTEGRATE AND FIRE MODEL
The basic circuit of an integrate and fire model consists of a capacitor and a resistor connected in parallel and driven by current I(t). Driving current I(t) can be broken down to two components I_R and I_C, where I(t)= I_R + I_C. I_R is the resistive input passing through resistor R and I_C charges the capacitor C. Therefore the membrane potential is assumed to integrate the input current[4]:

\[ C \frac{dV}{dt} = I(t) \]  

(1)

In the Integrate and fire model, the membrane potential characterizes the state of the neuron. Synaptic inputs arrive from other neurons via the associated synapses and are each weighted by their respective synaptic weights. The contributions of these synaptic inputs towards the membrane potential are excitatory or inhibitory. The synaptic input received by a neuron from other neurons is modeled either as injected current or as a change in membrane conductance. Models with injected current are called current synapse models where the summation of the synaptic inputs is linear. For a current synapse, the synaptic current is independent of the membrane potential. It is described as:

\[ I_{syn} = C_M \sum_{k=1}^{N_k} a_{EL,k} S_{EL,k}(t) + C_M \sum_{k=1}^{N_I} a_{IL,k} S_{IL,k}(t) \]  

(2)
Amplitudes, $a_{E,k} > 0$ and $a_{I,k} < 0$, are the potential changes due to a single synaptic event. $C_{M,E,k}$ and $C_{M,I,k}$ are the associated charge delivered to the neuron by the excitatory and inhibitory synaptic input. $S_{E,t}(t)$ and $S_{I,t}(t)$ are a series of input spikes to each synapse which describe the excitatory and inhibitory synaptic inputs given by:

$$S_{E,k}(t) = \sum t_{E,k} \delta(t - t_{E,k}), \quad S_{I,k}(t) = \sum t_{I,k} \delta(t - t_{I,k})$$

Here, $t_{E,k}$ and $t_{I,k}$ are respectively the times of the input synaptic spikes modeled usually as Poisson process with individual excitatory and inhibitory spiking rates. $S_{E}(t)$ and $S_{I}(t)$ denote the Poisson process associated with the $N_{E}$ excitatory and $N_{I}$ inhibitory synaptic inputs. $S_{E}(t) = \sum_{k} S_{E,k}(t)$, $S_{I}(t) = \sum_{k} S_{I,k}(t)$ with rate of spiking $\lambda_{E}$ and $\lambda_{I}$ respectively.

Conductance synapse models are modeled as a change in membrane potential and the summation of their synaptic inputs is nonlinear. Synaptic current is given by:

$$I_{S}(t) = C_{m} [V_{E} - V(t)] \sum_{k} g_{E,k} S_{E,k}(t) + C_{m} [v(t) - v(t)] \sum_{k} g_{I,k} S_{I,k}(t)$$  \hspace{1cm} (3)

$V_{E}$ and $V_{I}$ are the reversal potentials where $V_{I} < V_{E} < V_{th}$ . The equilibrium potential of the ion channels give rise to the reversal potentials. When the membrane potential passes through the corresponding reversal potential the direction of the associated current flow switches and gives rise to reversal potential[5,6]. Nonlinearity is introduced by the reversal potential into the summation of the individual synaptic inputs. The parameters $g_{E,k}, g_{I,k} > 0$ are taken as non-negative and represent the integrated excitatory and inhibitory conductance’s over the time course of the synaptic event divided by neural capacitance and are thus dimensionless.

A spike is generated by the neuron when the membrane potential reaches a threshold value of -50 to -55 mV. When the spiking takes place, the membrane potential follows a rapid trajectory and then returns to a hyperpolarized value which is below the threshold potential. So the integrate and fire model stipulates that a spike is generated whenever the membrane potential reaches the threshold value of $V_{th}$, $\lambda_{E}$ and $\lambda_{I}$ are the weights $a_{E}$ and $a_{I}$ respectively.

$$\mu = \lambda_{E} a_{E} - \lambda_{I} a_{I}$$  \hspace{1cm} (7)

The fluctuation in magnitude is determined to be:

$$\sigma^2 = \lambda_{E} a_{E} + \lambda_{I} a_{I}^2$$  \hspace{1cm} (8)

The drift parameter is taken as non-negative to ensure membrane potential reaches the threshold with probability one. The first passage time (FPT) can be defined as

$$T = \inf \{t \geq 0 | V(t) > V_{th}, V(0) < V_{th} \}$$  \hspace{1cm} (9)

which is found to be inverse Gaussian with probability density function

$$f(t) = \frac{(V_{th} - V(t))}{\sqrt{2\pi \sigma^2 t}} \exp \left[-\frac{(V_{th} - V(t) - \mu)^2}{2\sigma^2 t}\right]$$  \hspace{1cm} (10)

Where $t > 0$, $V_{th}$ corresponds to the threshold value and $V_{0}$ is the resting potential[7].

Integrate and fire models are useful for understanding how an ensemble of neuron interacts and how neurons sum large number of synaptic inputs. An ensemble of neurons is considered when a large number of nearby neurons with similar functions group together and fire together. There exists a degree of variability in the firing output of the integrate and fire neurons when they receive synaptic input. Thus neuronal response to multiple synaptic inputs occurs in two different modes of operations which depend on the balance between excitatory and inhibitory contributions. The spike train irregularity can be quantified using coefficient of variation ($C_{v}$). The coefficient of variation is the ratio of the standard deviation to the mean of the inter spike interval.

In the regular firing mode, the synaptic inputs received by the neurons attempt to charge the neuron to a potential which is above the threshold potential. But whenever the potential reaches the threshold value, it gets reset and starts charging again. So the timing of the action potential is determined by the charging rate of the neuron. When the excitatory input is stronger relative to the inhibitory input, the average membrane potential is depolarized than the spiking threshold of the model. This produces a regular spiking pattern. When the average membrane potential is more hyperpolarized than the spiking threshold of the model, an irregular spike train is generated. Certain advantages are...
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**I. INTRODUCTION**

Integrate and fire models play an important role in understanding information processing capabilities of a neuron. This model laid the framework for all modern discoveries in this field and helped scientists understand the concept of generation of action potential long before the mechanism for the generation of such an action potential was known. The model provides insights into a number of important questions and is also simple enough to provide analytical methods of solutions. The information transmitted from one neuron to another is contained in the sequence of action potential but the problem of neural coding has not yet been fully resolved. Further studies on this model can demonstrate how information processing is carried out in the human brain.

**II. SIMULATIONS**

Euler Maruyama method is applied to simulate stochastic neuronal models. When the membrane potential reaches threshold voltage $V_{th}$, a spike is generated and instantaneously the membrane potential is reset to its resting potential. Thereafter the membrane potential again evolves and reaches the threshold potential to generate another spike. Time interval between two consecutive spikes gives the Inter Spike Interval (ISI). Figure 1 represents the evolution of membrane potential and spike generation in IF model. Figure 2 shows the probability distribution function (PDF) of ISI of the IF model.

![Fig 1. Change in membrane potential in IF model with respect to time.](image1)

![Fig 2. Pdf of ISI in IF Model.](image2)

**TABLE 1: PARAMETERS USED**

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>VALUE</th>
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<tbody>
<tr>
<td>$N$</td>
<td>1000</td>
</tr>
<tr>
<td>$\mu$</td>
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</tr>
<tr>
<td>$\sigma$</td>
<td>0.2</td>
</tr>
<tr>
<td>$V_{th}$</td>
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**REFERENCES**


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